

# Chapter 3

## Perpendicular and Parallel Lines

# Section 5

## Using Properties of Parallel Lines

## GOAL 1: Using Parallel Lines in Real Life

When a team of rowers competes, each rower keeps his or her oars parallel to the adjacent rower's oars. If any two *adjacent* oars on the same side of the boat are parallel, does this imply that *any two* oars on that side are parallel? This question is examined below.

Example 1 justified Theorem 3.11, and you will prove Theorem 3.12 in Exercise 38.



## Example 1: Proving Two Lines are Parallel

Lines  $m$ ,  $n$ , and  $k$  represent three of the oars above.  $m \parallel n$  and  $n \parallel k$ . Prove that  $m \parallel k$ .

S

$m \parallel n, n \parallel k$

~~$\angle 1 \text{ cong. } \angle 2$~~

~~$\angle 2 \text{ cong. } \angle 3$~~

$\angle 1 \text{ cong. } \angle 3$

$m \parallel k$

R

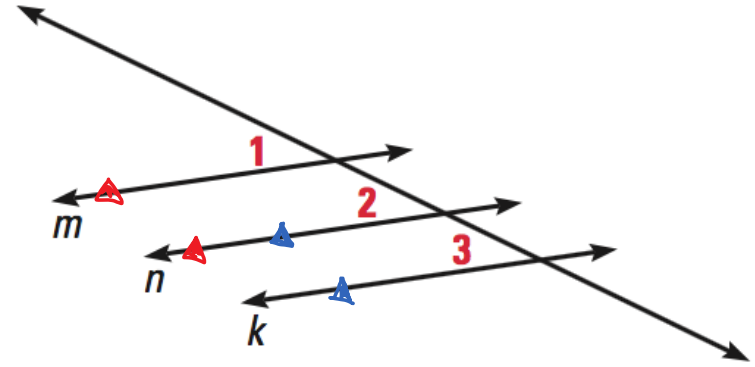
Given

Corr.  $\angle$ 's Theorem

Corr.  $\angle$ 's Theorem

Transitive

Corr.  $\angle$ 's CONVERSE

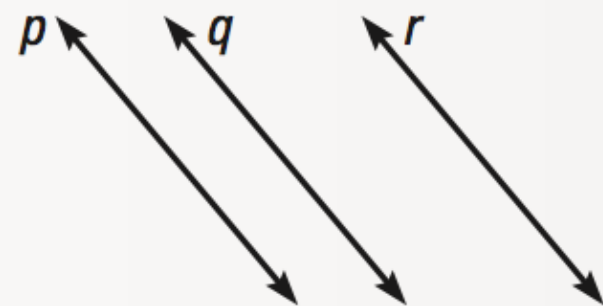


\*\*\*conclude lines parallel  $\rightarrow$  CONVERSE; conclude  $\angle$ 's congruent  $\rightarrow$  theorem\*\*\*

## THEOREMS ABOUT PARALLEL AND PERPENDICULAR LINES

### THEOREM 3.11

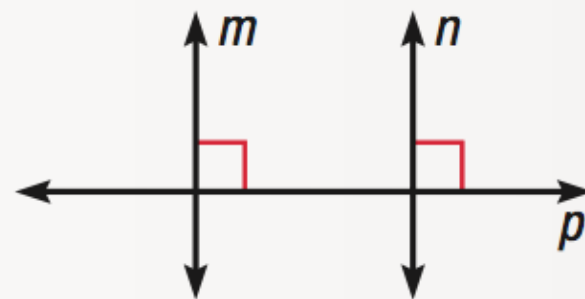
If two lines are parallel to the same line, then they are parallel to each other.



If  $p \parallel q$  and  $q \parallel r$ , then  $p \parallel r$ .

### THEOREM 3.12

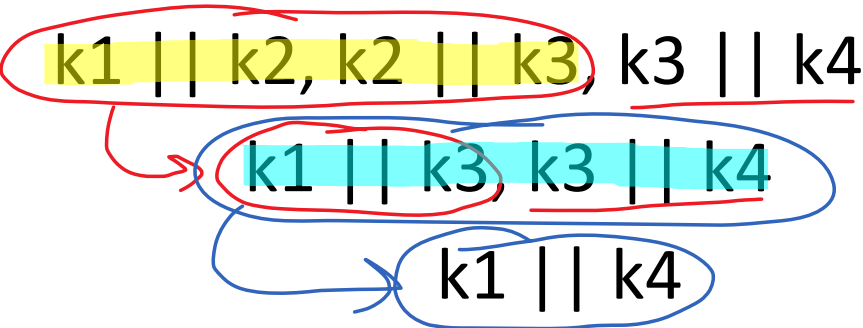
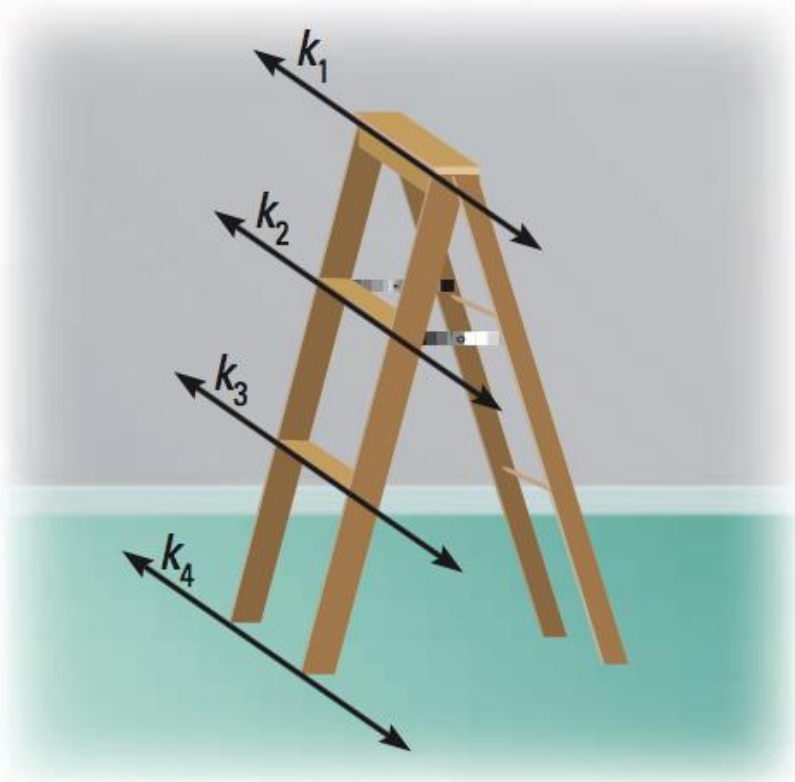
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.



If  $m \perp p$  and  $n \perp p$ , then  $m \parallel n$ .

# Example 2: Explaining Why Steps are Parallel

In the diagram at the right, each step is parallel to the step immediately below it and the bottom step is parallel to the floor. Explain why the top step is parallel to the floor.



(Thm. 3.11)

### Example 3: Building a CD Rack

You are building a CD rack. You cut the sides, bottom, and top so that each corner is composed of two  $45^\circ$  angles. Prove that the top and bottom front edges of the CD rack are parallel.

S

$$m\angle 1 = 45, m\angle 2 = 45, m\angle 3 = 45, m\angle 4 = 45$$

$$m\angle 1 + m\angle 2 = m\angle ABC; m\angle 3 + m\angle 4 = m\angle BCD$$

$$90 = m\angle ABC; 90 = m\angle BCD$$

$\angle ABC$  is a right  $\angle$ ;  $\angle BCD$  is a right  $\angle$

BA is perp. to BC; DC is perp. to BC

BA  $\parallel$  DC

R

Given

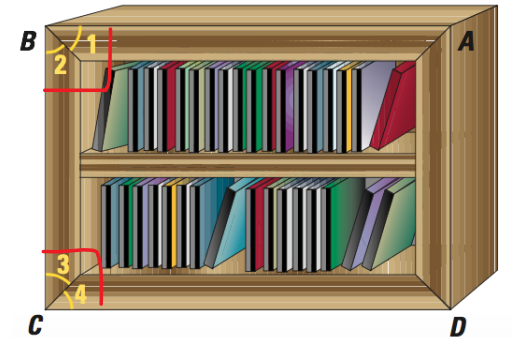
Angle Addition Postulate

Substitution/simplify

Def. of right angle

Def. of perp. Lines

3.12



## GOAL 2: Constructing Parallel Lines

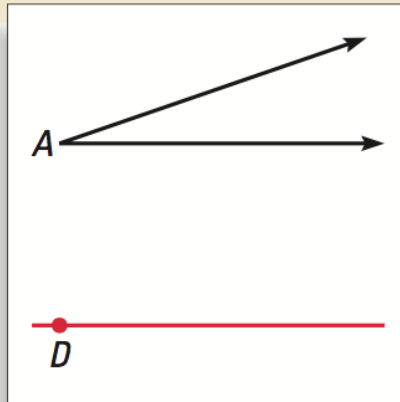
To construct parallel lines, you first need to know how to copy an angle.

### ACTIVITY

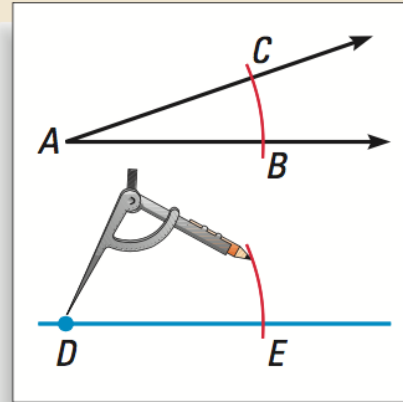
#### Construction

### Copying an Angle

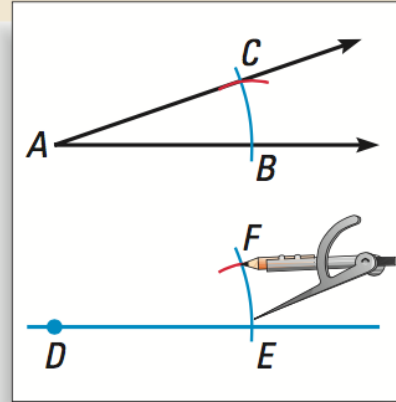
Use these steps to construct an angle that is congruent to a given  $\angle A$ .



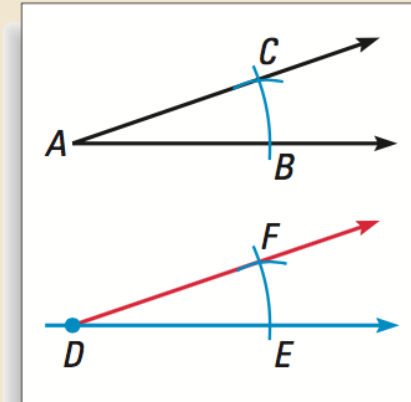
- 1 Draw a line. Label a point on the line  $D$ .



- 2 Draw an arc with center  $A$ . Label  $B$  and  $C$ . With the same radius, draw an arc with center  $D$ . Label  $E$ .



- 3 Draw an arc with radius  $BC$  and center  $E$ . Label the intersection  $F$ .



- 4 Draw  $\overrightarrow{DF}$ .  
 $\angle EDF \cong \angle BAC$ .



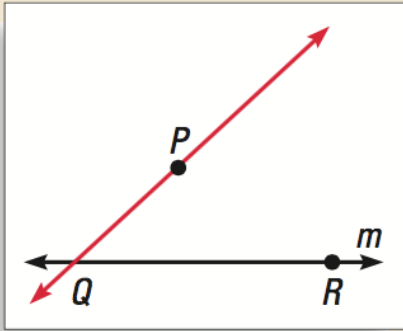
In Chapter 4, you will learn why the *Copying an Angle* construction works. You can use the *Copying an Angle* construction to construct two congruent corresponding angles. If you do, the sides of the angles will be parallel.

**ACTIVITY**

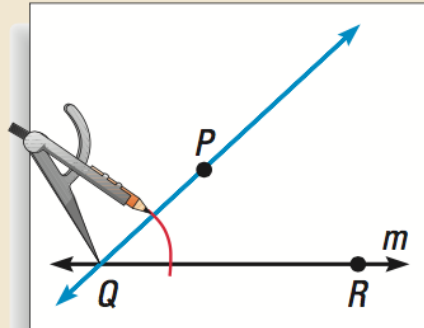
**Construction**

## Parallel Lines

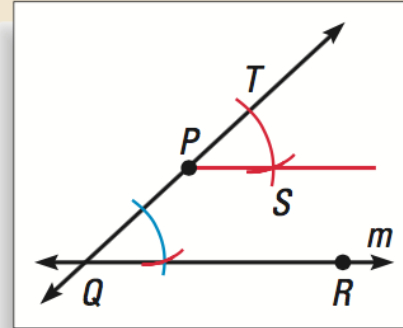
Use these steps to construct a line that passes through a given point  $P$  and is parallel to a given line  $m$ .



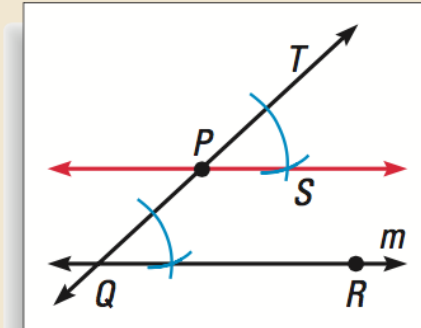
- 1 Draw points  $Q$  and  $R$  on  $m$ . Draw  $\overrightarrow{PQ}$ .



- 2 Draw an arc with the compass point at  $Q$  so that it crosses  $\overrightarrow{QP}$  and  $\overrightarrow{QR}$ .



- 3 Copy  $\angle PQR$  on  $\overrightarrow{QP}$  as shown. Be sure the two angles are corresponding. Label the new angle  $\angle TPS$  as shown.



- 4 Draw  $\overleftrightarrow{PS}$ . Because  $\angle TPS$  and  $\angle PQR$  are congruent corresponding angles,  $\overleftrightarrow{PS} \parallel \overleftrightarrow{QR}$ .